Gravity and the Quantum Vacuum Inertia Hypothesis
I. Formalized Groundwork for Extension to Gravity

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Abstract

It has been shown [1,2] that the electromagnetic quantum vacuum makes a contribution to the inertial mass, m_i, in the sense that at least part of the inertial force of opposition to acceleration, or inertia reaction force, springs from the electromagnetic quantum vacuum (see also [3] for an earlier attempt). Specifically, in the previously cited work, the properties of the electromagnetic quantum vacuum as experienced in a Rindler constant acceleration frame were investigated, and the existence of an energy-momentum flux was discovered which, for convenience, we call the Rindler flux (RF). The RF, and its relative, Unruh-Davies radiation, both stem from event-horizon effects in accelerating reference frames. The force of radiation pressure produced by the RF proves to be proportional to the acceleration of the reference frame, which leads to the hypothesis that at least part of the inertia of an object should be due to the individual and collective interaction of its quarks and electrons with the RF. We call this the *quantum vacuum inertia hypothesis*. We demonstrate that this approach to inertia is consistent with general relativity (GR) and that it answers a fundamental question left open within GR, viz. is there a physical mechanism that generates the reaction force known as weight when a specific non-geodesic motion is imposed on an object? Or put another way, while geometrodynamics dictates the spacetime metric and thus specifies geodesics, is there an identifiable mechanism for enforcing the motion of freely-falling bodies along geodesic trajectories? The quantum vacuum inertia hypothesis provides such a mechanism, since by assuming the Einstein principle of local Lorentz-invariance (LLI), we can immediately show that the same RF arises due to curved spacetime geometry as for acceleration in flat spacetime. Thus the previously derived expression for the inertial mass contribution from the electromagnetic quantum vacuum field is exactly equal to the corresponding contribution to the gravitational mass, m_g. Therefore, within the electromagnetic quantum vacuum viewpoint proposed in [1,2], the Newtonian weak equivalence principle, $m_i = m_g$, ensues in a straightforward manner. In the weak field limit it can then also be shown, by means of a simple argument from potential theory, that because of geometrical reasons the Newtonian gravitational force law must exactly follow. This elementary analysis however does not pin down the exact form of the gravitational theory that is required but only that it should be a theory of the metric type, i.e., a theory like Einstein's GR that can be interpreted as curvature of spacetime. While the present analysis shows that our previous quantum vacuum inertial mass analysis is consistent with GR, the extension of these two analyses to components of the quantum vacuum other than the electromagnetic component, i.e. the strong and weak vacua, remains to be done.

1. INTRODUCTION

Using the semiclassical representation of the electromagnetic quantum vacuum embodied in Stochastic Electrodynamics (SED), it has been shown that a contribution to the inertial mass, m_i, of an object must result from the interactions of the quantum vacuum with the electromagnetically-interacting particles (quarks and electrons) comprising that object [1,2]. Specifically, the properties of the electromagnetic quantum vacuum as measured in a Rindler constant acceleration frame were investigated, and the existence of an energy-momentum flux was discovered which, for convenience, we now call the Rindler flux (RF). The RF, and its relative, Unruh-Davies radiation, both stem from event-horizon effects in accelerating reference frames. Event horizons create an asymmetry in the quantum vacuum radiation pattern.

The force of radiation pressure on a massive object produced by the RF proves to be proportional to the acceleration of the reference frame attached to the object. This leads naturally to the hypothesis that at least part of the inertia of an object is due to the acceleration-dependent drag force that results from individual and collective interactions of the quarks and electrons in the object with the RF. For simplicity of reference, we refer to this concept, and an earlier derivation of a similar result using a completely different approach (involving a perturbation technique due to Einstein and Hopf on an accelerating Planck oscillator [3]), as the *quantum vacuum inertia hypothesis.*

SED is a theory that includes the effects of the electromagnetic quantum vacuum in physics by adding to ordinary Lorentzian classical electrodynamics a random fluctuating electromagnetic background constrained to be homogeneous and isotropic and to look exactly the same in every Lorentz inertial frame of reference [4,5]. This replaces the zero homogeneous background of ordinary classical electrodynamics. It is essential that this background not change the laws of physics when exchanging one inertial reference system for another. This translates into the requirement that this random electromagnetic background must have a Lorentz invariant energy density spectrum. The only random electromagnetic background with this property is one whose spectral energy density, $\rho(\omega)$, is proportional to the cube of the frequency, $\rho(\omega)d\omega \sim \omega^3 d\omega$. This is the case if the energy per mode is $\hbar\omega/2$ where ω is the angular frequency. (The $\hbar\omega/2$ energy per mode is of course also the minimum energy of the analog of an electromagnetic field mode: a harmonic oscillator.) The spectral energy density required for Lorentz invariance is thus identical to the spectral energy density of the zero-point field of ordinary quantum theory. For most purposes, including the present one, the zero-point field of SED may be identified with the electromagnetic quantum vacuum. However SED is essentially a classical theory since it presupposes only ordinary classical electrodynamics and hence SED also presupposes special relativity (SR).

According to the weak equivalence principle (WEP) of Newton and Galileo, inertial mass is equal to gravitational mass, $m_i = m_g$. If the quantum vacuum inertia hypothesis is correct, a very similar mechanism involving the quantum vacuum should also account for gravitational mass. This novel result, restricted for the time being to the electromagnetic vacuum component, is precisely what we show in §2 by means of formal but simple and straightforward arguments requiring physical assumptions that are uncontroversial and widely accepted in theoretical physics. In §3 the consistency of this argument with so-called metric theories of gravity (i.e. those theories characterized by spacetime curvature) is exhibited. In addition to the metric theory, *par excellance*, Einstein's GR, there is the Brans-Dicke theory and other less well known ones, briefly discussed by Will [6]. §4 briefly discusses a non-metric theory.

Nothing in our approach points to any new discriminants among the various metric theories. Nevertheless, our quantum vacuum approach to gravitational mass will be shown to be entirely consistent with the standard version of GR. Next, in §5, we take advantage of geometrical symmetries and present a short argument from standard potential theory to show that in the weak field limit a Newtonian inverse square force must result from our approach.

A new perspective on the origin of weight is presented in §6. In §7 we discuss an energetics aspect, related to the derivation presented herein, and resolve an apparent paradox. A brief discussion on the nature of the gravitational field follows in §8, but we infer that the present development of our approach does not provide any deeper or more fundamental insight than GR itself into the ability of matter to bend spacetime. We present conclusions in §9. A full development within GR is left for the accompanying article [7].

2. ON WHY THE ELECTROMAGNETIC VACUUM CONTRIBUTION TO GRAVITATIONAL MASS IS EXACTLY THE SAME AS FOR INERTIAL MASS

Table 1 compares the quantum vacuum inertia hypothesis with the standard view on mass. We intend to show – in this and a companion paper [7] – not only that the quantum vacuum inertia hypothesis is consistent with GR, but that it answers an outstanding question regarding a possible physical origin of the force manifesting as weight. We also intend to show that just as it becomes possible to identify a physical process underlying the **f**=m**a** postulate of Newtonian mechanics (as well as its extension to SR [1,2]), it is possible to identify a parallel physical process underlying the weak equivalence principle, $m_i = m_g$, viz. interaction of matter with the RF.

Within the standard theoretical framework of GR and related theories, the equality (or proportionality) of inertial mass to gravitational mass has to be assumed. It remains unexplained. As correctly stated by

Rindler [8], "the proportionality of inertial and gravitational mass for different materials is really a very mysterious fact." However here we show that – at least within the present restriction to electromagnetism – the quantum vacuum inertia hypothesis leads naturally and inevitably to this equality. The interaction between the electromagnetic quantum vacuum and the electromagnetically-interacting particles constituting any physical object (quarks and electrons) is identical for the two situations of acceleration with respect to constant velocity inertial frames or remaining fixed above some gravitating body with respect to freely-falling local inertial frames.

A related theoretical lucuna involves the origin of the force which manifests itself as weight. Within GR theory one can only state that deviation from geodesic motion results in a force which must be an inertia reaction force. We propose that it is possible in principle to identify a mechanism which generates such an inertia reaction force, and that in curved spacetime it acts in the same way as acceleration does in flat spacetime.

Begin by considering a macroscopic, massive, gravitating object, W, which is fixed in space and for simplicity we assume to be solid, of constant density, and spherical with a radius R, e.g. a planet-like object. At a distance $r >> R$ from the center of W there is a small object, w, that for our purposes we may regard as a point-like test particle. A constant force $\mathbf{f}$ is exerted by an external agent that prevents the small body w from falling into the gravitational potential of W and thereby maintains w at a fixed point in space above the surface of W. Experience tells us that when the force $\mathbf{f}$ is removed, w will instantaneously start to move toward W with an acceleration $\mathbf{g}$ and then continue freely falling toward W.

Next we consider a freely falling *local* inertial frame I_* (in the customary sense given to such a local frame [6]) that is instantaneously at rest with respect to w. At w proper time τ, that we select to be $\tau = 0$, object w is instantaneously at rest at the point $(c^2/g, 0, 0)$ of the I_* frame. The x-axis of that frame goes in the direction from W to w and, since the frame is freely falling toward W, at $\tau = 0$ object w appears accelerated in I_* in the x-direction and with an acceleration $\mathbf{g}_w = \hat{x}g$. As argued below, w is performing a uniformly-accelerated motion, i.e. a motion with a constant proper acceleration $\mathbf{g}_w$ as observed from any neighboring instantaneously comoving (local) inertial frame. In this respect we introduce an infinite collection of local inertial frames I_τ, with axes parallel to those of I_* and with a common x-axis which is that of I_*. Let w be instantaneously at rest and co-moving with the frame I_τ at w proper time τ. So the τ parameter representing the w proper time also serves to parametrize this infinite collection of (local) inertial frames. Clearly then, I_* is the member of the collection with $\tau = 0$, so that $I_* = I_{\tau=0}$. At the point in time of coincidence with a given I_τ, w is found momentarly at rest at the $(c^2/g, 0, 0)$ point of the I_τ frame. We select also the times in the (local) inertial frames to be t_τ and such that $t_\tau = 0$ at the moment of coincidence when w is instantaneously at rest in I_τ and at the aforementioned $(c^2/g, 0, 0)$ point of I_τ. Clearly as $I_{\tau=0} = I_*$ then $t_* = 0$ when $\tau = 0$. All the frames in the collection are freely falling toward W and when any one of them is instantaneously at rest with w it is instantaneously falling with acceleration $\mathbf{g} = -\mathbf{g}_w = -\hat{x}g$ with respect to w in the direction of W. It is not difficult to realize that w appears in those frames as uniformly accelerated and hence performing a hyperbolic motion with constant proper acceleration $\mathbf{g}_w$.

This situation is equivalent to that of an object w accelerating with respect to an ensemble of I_τ reference frames in the absence of gravity. In that situation, the concept of the ensemble of inertial frames, I_τ, each with an infinitesimally greater velocity (for the case of positive acceleration) than the last, and each coinciding instantaneously with an accelerating w is not difficult to picture. But how does one picture the analogous ensemble for w held stationary with respect to a gravitating body?

We are free to bring reference frames into existence at will. Imagine bringing a reference frame into existence at time $\tau = 0$ directly adjacent to w, but whereas w is fixed at a specific point above W, we let the newly created reference frame immediately begin free-falling toward W. We immediately create a replacement reference frame directly adjacent to w and let it drop, and so on. The ensemble of freely-falling local inertial frames bear the same relation to w and to each other as do the extended I_τ inertial frames used in the case of true acceleration of w.

For convenience we introduce a special frame of reference, S, whose x-axis coincides with those of the I_τ frames, including of course I_*, and whose y-axis and z-axis are parallel to those of I_τ and I_*. This frame S stays collocated with w which is positioned at the $(c^2/g, 0, 0)$ point of the S frame. For I_* (and for the I_τ frames) the frame S appears as accelerated with the uniform acceleration $\mathbf{g}_w$ of its point $(c^2/g, 0, 0)$. We will assume that the frame S is rigid. If so, the accelerations of points of S sufficiently separated from the w point $(c^2/g, 0, 0)$ are not going to be the same as that of $(c^2/g, 0, 0)$. This is not a concern however since

we will only need in all frames (I_τ, I_*, and S) to consider points in a sufficiently small neighborhood of the $(c^2/g, 0, 0)$ point of each frame

The collection of frames, I_τ, as well as I_* and S, correspond exactly to the set of frames introduced in [1]. The only differences are, first, that now they are all local, in the sense that they are only well defined for regions in the neighborhood of their respective $(c^2/g, 0, 0)$ space points; and second, that now I_* and the I_τ frames are all considered to be freely falling toward W and the S frame is fixed with respect to W. Similarly to [1], the S frame may again be considered to be, relative to the viewpoint of I_*, a *Rindler noninertial frame*. The laboratory frame I_* we now call the *Einstein laboratory frame*, since now the "laboratory" is local and freely falling. We call the collection of inertial frames I_τ the Boyer family of frames as he was the first to introduce them in SED [9].

The relativity principle as formulated by Einstein when proposing SR states that "all inertial frames are totally equivalent for the performance of all physical experiments."[6] Before applying this principle to the freely-falling frames I_* and I_τ that we have defined above it is necessary to draw a distinction between these frames and inertial frames that are far away from any gravitating body, such as W. The free-fall trajectories, i.e. the geodesics, in the vicinity of any gravitating body, W, cannot be parallel over any arbitrary distance owing to the fact that W must be of finite size. This means that the principle of relativity can only be applied locally. This was precisely the limitation that Einstein had to put on his infinitely-extended Lorentz inertial frames of SR when starting to construct GR [6,8,10].

We adopt the principle of local Lorentz invariance, (LLI) which can be stated, following Will [6], as "the outcome of a local nongravitational test experiment is independent of the velocity of the freely falling apparatus." A non-gravitational test experiment is one for which self-gravitating effects can be neglected. We also adopt the assumption of space and time uniformity, which we call the uniformity assumption (UA) and which states that the laws of physics are the same at any time or place within the Universe. Again, following Will [6] this can be stated as "the outcome of any local nongravitational test experiment is independent of where and when in the universe it is performed." We do not concern ourselves with physical cosmological theories that in one way or another violate UA, e.g. because they involve spatial or temporal changes in fundamental constants [11].

Locally, the freely falling local Lorentz frames which we now designate with a subscript L — $I_{*,L}$ and $I_{\tau,L}$ — are entirely equivalent to the I_* and I_τ extended frames of [1]. The free-falling Lorentz frame $I_{\tau,L}$ locally is exactly the same as the extended I_τ. Invoking the LLI principle we can then immediately conclude that the electromagnetic zero-point field, or electromagnetic quantum vacuum, that can be associated with $I_{\tau,L}$ must be the same as that associated with I_τ. From the viewpoint of the local Lorentz frames $I_{*,L}$ and $I_{\tau,L}$ the body w is undergoing uniform acceleration and therefore for the same reasons as presented in [1] an acceleration-dependent drag force arises.

These formal arguments demonstrate that the analyses of of [1,2] which found the existence of a RF in an accelerating reference frame translate and correspond exactly to a reference frame fixed above a gravitating body. In the same manner that light rays are deviated from straight-line propagation by a massive gravitating body W, the other forms of electromagnetic radiation, including the electromagnetic zero-point field rays (in the SED approximation) are also deviated from straight-line propagation. Not surprisingly this creates an anisotropy in the otherwise isotropic electromagnetic quantum vacuum. This is the origin of the RF in the gravitational case.

In [1] we interpreted the drag force exerted by the RF as the inertia reaction force of an object that is being forced to accelerate through the electromagnetic zero-point field. Accordingly, in the present situation, the associated nonrelativistic form of the inertia reaction force should be

$$\mathbf{f}_*^{zp} = -m_i \mathbf{g}_w \tag{1}$$

where $\mathbf{g}_w$ is the acceleration with which w appears in the local inertial frame I_*. As shown in [1] the coefficient m_i is

$$m_i = \left[\frac{V_0}{c^2} \int \eta(\omega) \frac{\hbar \omega^3}{2\pi^2 c^3} d\omega \right] \tag{2}$$

where V_0 is the proper volume of the object, c is the speed of light, $\hbar$ is Planck's constant divided by 2π and $\eta(\omega)$, where $0 \leq \eta(\omega) \leq 1$, is a function that spectralwise represents the relative strength of the interaction

between the zero-point field and the massive object which acts to oppose the acceleration. If the object is just a single particle, the spectral profile of $\eta(\omega)$ will characterize the electromagnetically-interacting particle. It can also characterize a much more extended object, i.e. a macroscopic object, but then the $\eta(\omega)$ will have much more structure (in frequency). We should expect different shapes for the electron, a given quark, a composite particle like the proton, a molecule, a homogeneous dust grain or a homogeneous macroscopic body. In the last case the $\eta(\omega)$ becomes a complicated spectral opacity function that must extend to extremely high frequencies such as those characterizing the Compton frequency of the electron and even beyond.

Now, however, what appears as inertial mass, m_i, to the observer in the local $I_{*,L}$ frame is of course what corresponds to gravitational mass, m_g, and it must therefore be the case that

$$m_g = \left[\frac{V_0}{c^2}\int \eta(\omega)\frac{\hbar\omega^3}{2\pi^2 c^3}d\omega\right].$$ (3)

As done in [1], Appendix B, it can be shown that the right hand side indeed represents the energy of the electromagnetic quantum vacuum enclosed within the object's volume and able to interact with the object as manifested by the $\eta(\omega)$ coupling function. A more thorough, fully covariant development can also be implemented to show that the force expression of eqn. (1) can be extended to the relativistic form of the inertia reaction force as in [1] , Appendix D. (This development also served to obtain the final form of m_i given above in eqn. (3) eliminating a spurious 4/3 factor.) * Summarizing what we have shown in this section is that if a force **f** is applied to the w body just large enough to prevent it from falling toward the body W, then in the non-relativistic case that force is given by

$$\mathbf{f} = -m\mathbf{g}$$ (4)

where we have dropped the nonessential subscripts i and g and superscripts, because it is now clear that $m_i = m_g = m$ follows from the quantum vacuum inertia hypothesis.

3. CONSISTENCY WITH EINSTEIN'S GENERAL RELATIVITY

The statement that $m_i = m_g$ constitutes the weak equivalence principle (WEP). Its origin goes back to Galileo and Newton, but it now appears, as shown in the previous section, that this principle is a natural consequence of the quantum vacuum inertia hypothesis. The strong equivalence principle (SEP) of Einstein consists of the WEP together with LLI and the UA. Since the quantum vacuum inertia hypothesis and its extension to gravity allow us to obtain the WEP assuming LLI and the UA, this approach is consistent with all theories that are derived from the SEP. In addition to GR, the Brans-Dicke theory is derived from SEP as are other other lesser known theories [6], all distinguished from each other by various particular assumptions.

All theories that assume the SEP are called *metric theories*. They are characterized by the fact that they contemplate a bending of spacetime associated with the presence of matter. Two important consequences of the LLI-WEP-UA combination are that light bends in the presence of matter and that there is a gravitational Doppler shift. Since the quantum vacuum inertia hypothesis is consistent with this same combination, it would also require that light bends in the presence of gravitational fields. This can, of course, be interpreted as a change in spacetime geometry, the standard interpretation of GR.

4. A RECENT ALTERNATIVE VACUUM APPROACH TO GRAVITY

The idea that the vacuum is ultimately responsible for gravitation is not new. It goes back to a proposal of Sakharov [12] based on the work of Zeldovich [13] in which a connection is drawn between Hilbert-Einstein

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* We use this opportunity to correct a minor transcription error that appeared in the printed version of Ref. [1]. In Appendix D, p. 1100, the minus sign in eqn. (D8) is wrong. It should read

$$\eta^\nu\eta_\nu = 1$$ (D8)

and the corresponding signature signs in the line just above eqn. (D8) are the opposite of what was written and should instead read $(+ - - -)$.

action and the quantum vacuum. This leads to a view of gravity as "a metric elasticity of space" (see Misner, Thorne and Wheeler [14] for a succinct review of this concept). Following Sakharov's idea and using the techniques of SED, Puthoff proposed that gravity could be construed as a form of van der Waals force [15]. Although interesting and stimulating in some respects, Puthoff's attempt to derive a Newtonian inverse square force of gravity proves to be unsuccessful [16,17,18,19].

An alternative approach has recently been developed by Puthoff [20] that is based on earlier work of Dicke [21] and of Wilson [22]: a polarizable vacuum model of gravitation. In this representation, gravitation comes from an effect by massive bodies on both the permittivity, ϵ_0, and the permeability, μ_0, of the vacuum and thus on the velocity of light in the presence of matter.

This is clearly an alternative theory to GR since it does not involve curvature of spacetime. So it is not a metric theory of gravity. It can be seen that it does not entail the SEP of Einstein. On the other hand, since spacetime curvature is by definition inferred from light propagation in relativity theory, the polarizable vacuum gravitation model may be labelled a pseudo-metric theory of gravitation since the effect of variation in the dielectric properties of the vacuum by massive objects on light propagation are approximately equivalent to GR spacetime curvature as long as the fields are sufficiently weak.

In the weak-field limit, the polarizable vacuum model of gravitation duplicates the results of GR, including the classic tests (gravitational redshift, bending of light near the Sun, advance of the perihelion of Mercury). Differences appear in the strong-field regime, which should lead to interesting tests.

5. DERIVATION OF NEWTON'S LAW OF GRAVITATION

Our approach allows us to derive a Newtonian form of gravitation in the weak-field limit based on the quantum vacuum inertia hypothesis, local Lorentz invariance and geometrical considerations. In [1] we showed how an asymmetry that appears in the radiation pattern of the electromagnetic quantum vacuum, when viewed from an accelerating reference frame, leads to the appearance of a non-zero momentum flux, the Rindler Flux (RF), opposing any accelerating object. Individual and collective interaction between the electromagnetically-interacting particles (quarks and electrons) comprising a material object and the RF generates a reaction force that may be identified with the inertia reaction force as it has the right form for all velocity regimes. In particular in the low velocity limit it is exactly proportional to the acceleration of the object.

In §2 we showed from formal arguments based on the principle of local Lorentz invariance (LLI) that an exactly equivalent force must originate when an object is effectively accelerated with respect to free-falling Lorentz frames by virtue of being held fixed above a gravitating body, W. We infer that the presence of a gravitating body must distort the electromagnetic quantum vacuum in exactly the same way at any given point as would the process of acceleration such that $\mathbf{a} = -\mathbf{g}$. Simple geometrical arguments [23] now suffice to show that the gravitational force can only be the Newton inverse-square law with distance (in the weak field limit).

It has been shown (§2) that (outside W) the $\mathbf{g}$ field of eqn. (4) generated by W has to be central, i.e., centrally distributed with spherical symmetry around W. It has to be radial, with its vectorial direction parallel to the corresponding radius vector $\mathbf{r}$ originating at the center of mass of W where we locate the origin of coordinates. The spherical symmetry implies that $\mathbf{g}$ is radial in the direction $-\hat{\mathbf{r}}$, and depends only on the r-coordinate.

The field is clearly generated by mass. A simple symmetry argument, here omitted for brevity [24], shows that indeed if we scale the mass M of W by a factor α then the resulting $\mathbf{g}$ must be of the form $\alpha\mathbf{g}$. If M goes to zero, $\mathbf{g}$ disappears. The mass M must be the source of field lines of $\mathbf{g}$, and these field lines can be discontinuous only where mass is present. The field lines can be neither generated nor destroyed in free space. Since $\mathbf{g}$ is the force on a unit mass, we must expect that $\mathbf{g}$ behaves as a vector, and specifically that $\mathbf{g}$ follows the laws of vector addition. Namely, if two masses M_1 and M_2 in the vicinity of each other generate fields $\mathbf{g}_1$ and $\mathbf{g}_2$ respectively, the resulting $\mathbf{g}$ at any given point in space should be the linear superposition

$$\mathbf{g} = \mathbf{g}_1 + \mathbf{g}_2. \tag{5}$$

Finally, from the argument that leads to eqn. (4) we can see that the field $\mathbf{g}$ must be unbounded, extending essentially to infinity.

With all of these considerations, clearly the lines of $\mathbf{g}$ must obey the continuity property outside W. If there is no mass present inside a volume, V, enclosed by a surface S, we expect that

$$0 = \oint_{S(V)} \mathbf{g} \cdot \mathbf{n} \, dS = \int_V \nabla \cdot \mathbf{g} \, dV, \tag{6}$$

but since V is arbitrary this tells us that outside the massive body W

$$\nabla \cdot \mathbf{g} = 0, \quad r > R . \tag{7}$$

In the presence of our single massive body, W, but outside that body, $\mathbf{g}$ is radial from the center of mass, and therefore from eqn. (7) we conclude that

$$\mathbf{g} \sim -\hat{\mathbf{r}} \frac{1}{r^2} \tag{8}$$

where the restriction $r > R$ is hereafter understood. Since the field $\mathbf{g}$ is also proportional to the mass M which is its origin, we conclude that $\mathbf{g}$ must be of the form

$$\mathbf{g} = -\hat{\mathbf{r}} \frac{GM}{r^2}. \tag{9}$$

where G is a proportionality constant and from eqn. (4) we have that

$$\mathbf{f} = -\hat{\mathbf{r}} \frac{GMm}{r^2}. \tag{10}$$

which is Newton's law of gravitation. It is remarkable that after finding the central and radial charater of $\mathbf{g}$ by means of the vacuum approach of our quantum vacuum inertia hypothesis, one can immediately obtain Newtonian gravitation, an endeavour keenly but unsuccessfully pursued for quite some time from the viewpoint of the vacuum fields [12,13,14] and in particular of SED [15,16,17,18,19]. In this last case (SED), it was proposed that gravity was a force of the van der Waals form, a view which has been shown to be unsuccessful [19].

6. ON THE ORIGIN OF WEIGHT

We have established that the quantum vacuum inertia hypothesis leads to a force in eqn. (10) which adequately explains the origin of weight in a Newtonian view of gravitation. How is this consistent with the geometrodynamic view of GR? Geometrodynamics specifies the effect of matter and energy on an assumed pliable spacetime metric. That defines the geodesics which light rays and freely-falling objects will follow. However there is nothing in geometrodynamics that points to the origin of the inertia reaction force when geodesic motion is prevented, which manifests in special circumstances as weight.

Geometrodynamics merely assumes that deviation from geodesic motion results in inertial forces. That is, in fact, true, but as stated is devoid of any physical insight as discussed in detail in [25]. What we have shown above is that an identical asymmetry in the quantum vacuum radiation pattern will arise due to either true acceleration or to effects on light propagation by the presence of gravitating matter. That identical asymmetry leads to identical non-zero RFs in the electromagnetic zero-point field, which generate identical forces. In the case of true acceleration, the resulting force is the inertia reaction force. In the case of being held stationary in a non-Minkowski metric, the resulting force is the weight, which is also the enforcer of geodesic motion for freely-falling objects.

7. A PHANTOM ANOMALY

Following the reasoning of §6 one would conclude that at every point of fixed r above W there is an inflowing RF. This would seem to imply a continuous energy flux toward and into W, an apparently paradoxical situation consisting of quantum vacuum-originating energy streaming toward W from all directions in space.

The resolution to this apparent paradox comes from the realization that we are naively summing energy flows from incompatible reference frames. Two observers at the same r but 180 degrees apart will report energy flows in opposite directions. These energy flows are locally true, but they come from oppositely-directed reference frames. Only in a freely-falling frame in which all of the energy fluxes could simultaneously be properly defined could we legitimately simply sum them up. However it is mathematically improper and non-physical to add up over vectors defined in different reference frames, which moreover in the present case can be said to even be incompatible.

For the sake of definiteness, let us locate ourselves and w not very far away from the surface of W, and for simplicity let W be perfectly spherical with radius R and homogeneous in density. Let the test body w be, as before, much smaller than W and assume that it is prevented from falling toward W by a supporting force $\mathbf{f}$ as in eqn. (4).

We need only realize two facts. The analysis leading to the RF that appears for w is only consistent for a given freely-falling LLI frame or at most for the Boyer family of frames I_τ, where $I_{\tau=0}=I_*$, along the particular straight radius vector going from W to w. (For simplicity of notation we drop the L-index subscript that indicates the locality of the frames as there is no source of confusion at this point.)

For example, consider a small body w' (resting at the $(c^2/g, 0, 0)$ point of S') and associated set of frames I'_τ and I'_* falling freely toward W and defined exactly the same as I_τ and I_* were defined with respect to w but whose radius vector of free fall toward W is along the widely different direction that goes from W to w'. As occurs with the observers of unprimed frames I_τ and I_*, observers in the primed frames I'_τ and I'_* also claim there appears a RF in S' along the radial direction or x'-axis and hitting the body w' in the direction pointing toward W. But for the primed frame, the unprimed frame's direction of fall toward W is not a direction of any net RF. The reverse conclusion also holds. For the unprimed frame there is no net RF along the radial direction (x'-axis) of the primed frame.

Therefore all this shows that those RFs only can be inferred as such from a selectively restricted class of LLI frames. The only way we could produce a consistent conclusion about this odd thermodynamics situation is if we could find a single freely-falling LLI frame from which we could simultaneously make a consistent conclusion about the nature of all such RFs, i.e. a frame from which we can define simultaneously and hence be able to add up over all those different RFs. Fortunately there exists such a frame.

Consider W as the sphere of the Earth. As one goes more and more deeply inside, the strength of $\mathbf{g}$ decreases. So freely-falling frames ever deeper within the Earth fall with ever smaller accelerations, and at the exact center of mass of the Earth $\mathbf{g}$ is exactly zero. We can actually quantify this in full precision since we have already discovered that the gravitational force must go as r^{-2}.

There is one single freely-falling LLI frame that is a member of all radial families of freely-falling frames and that is the LLI frame exactly at the center of the Earth that we shall call C. From that frame C we can observe and draw consistent conclusions about the nature of all the RFs along all possible radial directions toward the center of W. But C is not only freely falling with exactly zero acceleration, but also has always been at rest at the center of the Earth. Furthermore conclusions made from this freely-falling frame can be universal since it has zero acceleration and, at least from this limited perspective, can be extended indefinitely, i.e. need not be strictly local.

Then for that frame, C, we can consistently look in all possible radial directions. Consider the direction from the center of W to w. Since w is at a fixed distance from the center of W and this is fixed and not moving with respect to an observer in C, there cannot be any RF due to quantum vacuum radiation of C impinging on w, because w does not appear accelerated as viewed from C. So the RF disappears along the particular radial direction. But for that matter, it disappears along all radial directions from the center of W and the paradox is resolved. The only single frame from which a consistent conclusion referring to all RFs along all possible radial directions from the center of W can be drawn yields that such RFs exactly vanish. The observer in C does not infer any net radiation influx toward W from any direction in space.

We also note that any observer in orbit around W will detect no RF. Such an observer will detect a RF should he attempt to change orbits, i.e. deviate from geodesic motion. Indeed, he must apply a force to overcome the effect of the RF when he attempts to change orbits.

8. DISCUSSION

In light of what has been proposed herein, what can we elucidate about the nature of the gravitational field? In the low fields and low velocities version, or the Newtonian limit, we have seen above that gravity

manifests itself as the attractive force per unit mass, **g**, of eqn. (9) that pulls any massive test body present at a given point in space towards the body W that originates the field. Since we assumed the Einstein LLI principle and from this derived the WEP, this, together with the very natural UA of invariance in the laws of physics throughout universal spacetime, lead us to the Einstein SEP which necessarily implies the spacetime bending representation of the generalized gravitational field [6,8].

A simple thought experiment (Einstein's lift) immediately shows that light rays propagate along geodesics, and more specifically along null geodesics [8,14,26]. The spacetime bending is dramatically evident when a light ray goes from one side to the other of the freely falling elevator. For the observer attached to the elevator's frame that indeed acts as a LLI frame, the light ray propagates in a straight line from one side to the other of the elevator. But for the stationary observer who sees the elevator falling with acceleration **g**, the light ray bends along a path that locally is seen as a parabolic curve. Undoubtedly the most natural explanation for the stationary observer is that spacetime bends and therefore the association that this bending is a manifestation of the gravitational field of W, or rather that this bending of spacetime is the gravitational field itself [26].

Starting from the above fact taken as a given, the various metric theories proceed from there to formulate their equations. In the version of Brans and Dicke a scalar-tensor field is assumed. In Einstein's GR only a tensor field is proposed. Following this maximally simplistic proposal and guided by general considerations of general covariance (the need of arbitrary coordinates and tensor laws) Einstein was led to his field equations in the presence of matter. And then GR naturally unfolded [6,8,14,26].

We have shown here that our inertia proposal of [1] leads us, when limited by the LLI principle, to the metric theories and therefore that it is consistent with those theories and in particular with Einstein's GR. In addition, there is the following interesting feature of our proposal.

From our analysis in [1], and in particular in Appendix B of [1], it was made clear that within the quantum vacuum inertia hypothesis there proposed, the mass of the object, m, could be viewed as the energy in the equivalent vacuum electromagnetic zero-point field captured within the structure of the object and that readily interacted with the object. This view properly and accurately matched with the complementary view, thoroughly exposed in other parts of the paper [1], that presented inertia as the result of a vacuum reaction effect, a kind of drag force exerted by the vacuum field on accelerated objects. Quantitatively, both approaches lead to exactly the same inertial mass and moreover they were partly complementary. Both viewpoints were needed. One could not exist without the other. They were the two sides of the same coin.

The question is now why massive objects, when freely falling, also follow geodesic paths. The tempting view suggested here is that, as massive bodies (according to our analysis of [1], Appendix B) have a mass that is made of the vacuum electromagnetic energy contained within their structure and that readily interacts with such structure, it is no surprise that geodesics are their natural path of motion during free fall. Electromagnetic radiation has been shown by Einstein to follow precisely geodesic paths. The only difference now is that, as the radiation stays within the accelerated body structure and is contained within that structure and thereby its energy center moves subrelativistically, these geodesics are just time-like and not null ones as in the case of freely propagating light rays.

We illustrate this with an example. Imagine a freely-falling electromagnetic cavity with perfectly reflecting walls of negligible weight, so that all the weight is due to the enclosed radiation. A simple plane wave mode decomposition shows that although individual wavetrains do still move at the speed of light, the center of energy of the radiation inside the cavity moves subrelativistically as the wavetrains reflect back and forth. The wavetrains do indeed propagate along null geodesics, but the center of energy propagates only along a time-like geodesic.

Neither our approach nor the conventional presentations of GR for that matter, can offer a physical explanation of the mechanism of the bending of spacetime as related to energy density. Misner, Thorne and Wheeler [14] present six different proposed explanations. The sixth is the one we already mentioned due to Sakharov [12,13] which starts from general vacuum considerations. As our approach starts also from vacuum considerations, it naturally fits better the concept of the conjecture of Sakharov [12] and Zeldovich [13] than the other proposals but it is not inconsistent with any of them. In particular the strictly formal proposal of Hilbert [14] that introduces the so-called Einstein-Hilbert Action, is also at the origin of the Sakharov proposal. We plan to devote more work to exploring the connection of our inertia [1,2] and gravity approach to the approach proposed by Sakharov [12]. This we leave for a future publication.

9. CONCLUSIONS

The principal conclusions of this paper are:

(1) *Identity of inertial mass with gravitational mass, $m_i = m_g$.* It has been shown that the approach of [1] contains this peculiar feature that, so far and as we know, has never been explained. Rindler [7] calls this feature "a very mysterious fact," as indeed it has been up to the present. We expect with this work to have shed some light on this peculiar feature.

(2) *Consistency of the quantum vacuum inertia hypothesis with Einstein's GR.* We have already commented above, in particular in §7, on this interesting feature presented in §3 that puts the vacuum inertia approach of [1] within the mainstream thought of contemporary gravitational theories, specifically within that of theories of the metric type and in particular in agreement with GR. The full GR development of gravitation within the scope of the quantum vacuum inertia hypothesis is presented in [7].

(3) *Newton's gravitational law from the quantum vacuum inertia hypothesis.* By means of a simple argument based on potential theory we show how to obtain in a natural way Newton's inverse square force with distance from our vacuum approach to inertia of [1]. The simplicity of our approach contrasts with previous atempts to accomplish this within the framework of SED theory [15,16,17,18,19].

(4) *Origin of weight and a physical mechanism to enforce motion along geodesic trajectories for freely-falling objects.* We have shown how this approach to inertia answers a fundamental question left open within GR, viz. is there a physical mechanism that generates the inertia reaction when non-geodesic motion is imposed on an object and which can manifest specifically as weight. Or put another way, while geometrodynamics dictates the spacetime metric and thus specifies geodesics, is there an identifiable mechanism for enforcing motion along geodesic trajectories? The quantum vacuum inertia hypothesis represents a significant first step in providing such a mechanism.

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TABLE 1.

Comparison of Standard View of Mass and Quantum Vacuum Inertia Hypothesis

Standard View of Mass	Quantum Vacuum Inertia Hypothesis
INERTIA REACTION FORCE	
accelerating object experiences event horizon	accelerating object experiences event horizon
event horizon promotes some quantum vacuum (QV) energy to "real photons"	event horizon promotes some quantum vacuum (QV) energy to "real photons"
accelerating object experiences QV "real photons" as Unruh-Davies radiation	accelerating object experiences QV "real photons" as Unruh-Davies radiation *plus Rindler flux*
Higgs field can generate mass-energy for quarks and electrons	Higgs field can generate mass-energy for quarks and electrons
inertia reaction force arises from intrinsic property of matter	inertia reaction force is an acceleration-dependent drag force resulting from the Rindler flux
f=m**a** is postulated	**f**=m**a** ensues from hypothesis
GRAVITATIONAL FORCE/WEIGHT	
gravitating body determines geodesics	gravitating body determines geodesics
light rays and freely-falling objects follow geodesics	light rays and freely-falling objects follow geodesics
weight of stationary object is an inertia reaction force due to deviation from geodesic	weight of stationary object is an inertia reaction force due to deviation from geodesic
inertia reaction force arises from intrinsic property of matter	inertia reaction force is a metric-dependent drag force resulting from the Rindler flux
$m_{inertial} = m_{gravitational}$ is postulated	$m_{inertial} = m_{gravitational}$ ensues from hypothesis